

The Holographic RG flow to conformal and non-conformal theory*

M. Petrini and A. Zaffaroni

M.P.: Theoretical Physics Group,
Imperial College, London SW7 2BZ, UK.
A.Z.: INFN, Sezione di Milano,
via Celoria 16, 20133, Milano, Italy.

ABSTRACT: We review some aspects of the AdS supergravity description of RG flows. The case of a flow to an IR CFT can be rigorously studied within the framework of supergravity. Here we discuss various central charges of the conformal theory (included the usually neglected ones) and we compare them with QFT expectations. The case of flows to non-conformal theories is more problematic in that one usually encounters a naked singularity. We mainly focus on the flow to an IR $N=1$ super Yang-Mills theory. We discuss the properties of the solution and we briefly comment on the fate of the singularity. We also compare the supergravity results with the expectations of an $N=1$ SYM at strong coupling.

1. Introduction

The AdS/CFT correspondence has deserved some surprises when extended outside the realm of strictly conformally invariant theories. The study of the supergravity dual of RG flows has flourished, both in the concrete application to SYM theories and in a general setting [1]–[14]. Asymptotically AdS_{d+1} backgrounds, breaking the full $O(d,2)$ invariance but preserving at least d -dimensional Poincaré invariance, describe RG flows for a d -dimensional CFT. These supergravity solutions with an asymptotic AdS region have a double QFT interpretation: deformations of an UV fixed point versus the same theory in a different vacuum [15, 16]. Both cases have been extensively studied. Many results have been obtained upon reduction to a $d+1$ -dimensional effective theory, where the RG flow can be studied in terms of a theory of scalar fields coupled to gravity. In this simple set-up, the RG flows are identified as domain-walls interpolating between AdS_{d+1} vacua (or approaching infinity on one side), and general results are very easy to obtain. The cor-

respondence defines a *holographic* scheme, where beta and c -functions have a natural definition. A c -theorem, for example, can be easily proven [1, 8]. Moreover, it is possible to obtain the quantum field theory RG equations from supergravity [14]¹.

The study of RG flows between CFTs (at large N and strong coupling) can be rigorously performed using supergravity. The phase space of massive deformations of the $N=4$ SYM theory has been thoroughly investigated and several IR fixed points have been found [1, 2, 3, 4, 8]. The results are on solid grounds because supergravity is valid all along the RG flow. Still problematic is the precise mapping of some QFT couplings to supergravity quantities. For example, it is still unclear what in supergravity corresponds to the running of the gauge coupling.

Most of the unsolved problems concern the flows to non-conformal theories, where supergravity is invalidated by a (typically naked) singularity in the IR region of the flow. Solutions

¹Notice that the *holographic* beta and c -functions do not need to coincide with analogous functions defined in schemes that are more natural from the QFT point of view [17].

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flowing to infinity for a generic $5d$ -Lagrangian are certainly a dense set in the space of solutions. The full recipe for selecting the *physical* ones is still unclear². The distinction between deformations and vacua of an UV fixed point helps but does not solve the problem. Supersymmetric and supersymmetric-inspired solutions however are uniquely selected because the equations of motion can be reduced to first order ones [8, 19]. $N=4$ Coulomb branch solutions have been studied in [9, 10, 20]. Here we focus on the flow to $N=1$ SYM. Despite the singularity, we obtain a good qualitative agreement with quantum field theory expectations already at the level of supergravity.

Since singularities are apparently unavoidable in interesting supergravity solutions, it is mandatory to understand their fate in the full string theory, where they must be resolved. Available options are the chance that the singularity is an artifact of the dimensional reduction to 5 dimensions, mechanisms such that proposed in [21] and, more generally, some help from string corrections.

The supergravity solutions with an asymptotic AdS region certainly have many other applications. Relaxing the d -Poincaré invariance, we have examples of RG flow due to finite temperature. This is indeed the firstly proposed method for discussing non-conformal theories from AdS [22] and the one not suffering from unpleasant singularities. Cutting the AdS-boundary, we can describe CFTs coupled to gravity and make contact with the large extra-dimension scenario [23]. We will not discuss this issue here, but we simply notice that singular solutions have been recently considered in this context.

2. RG Flow from 5d Supergravity

In general, we interpret the $(d+1)$ -th coordinate y of AdS_{d+1} as an energy scale [24, 25]. RG flows between CFTs then correspond to type II or M-theory supergravity solutions interpolating (along y) between $\text{AdS}_{d+1} \times_W H$ vacua.

The very first example of RG flow in the AdS/CFT correspondence is manifest in the multi-

²A criterion for selecting physical solutions has been recently proposed in [18].

centre supergravity solution for D3-branes [24]. This represents the Coulomb branch of $N=4$ SYM. Given two sets of N and M branes at different points, the near-horizon geometry is AdS_5 with radius $\sim \sqrt{N+M}$ far from both sets of branes, and AdS_5 with radius $\sim \sqrt{N}$ near one set. In QFT this is the RG flow between the $U(N+M)$ $N=4$ CFT in the UV, where the Higgs VEVs can be neglected, and the $U(N)$ $N=4$ CFT in the IR. A more sophisticated example was found in [26]. A supergravity solution interpolating between $\text{AdS}_5 \times S^5/Z_2$ and $\text{AdS}_5 \times T^{1,1}$ was also interpreted on the QFT side as a RG flow between CFTs. It is a supersymmetric massive deformation of the $N=2$ $SU(N) \times SU(N)$ theory corresponding to a Z_2 orbifold of $N=4$ SYM which flows to an $N=1$ IR fixed point. Many successful checks of this interpretation have been performed [26, 27, 28, 29].

However, interpolating $10d$ backgrounds are difficult to find. Sometimes dimensional reduction to 5 dimensions helps.

The RG flow has a natural description in $5d$. Consider a certain UV CFT and suppose we have the corresponding $5d$ Lagrangian and that it contains all the fields/modes we are interested in. The effective $5d$ Lagrangian we need is just the most general Lagrangian for scalars coupled to gravity

$$L = \sqrt{-g} \left[-\frac{R}{4} + \frac{1}{2} g^{IJ} \partial_I \lambda_a \partial_J \lambda_b G^{ab} + V(\lambda) \right]. \quad (2.1)$$

The scalars λ_a can either be the *massless* modes or Kaluza-Klein modes of the compactification to 5 dimensions. The form of the potential depends on the particular case we are considering. We may have, for example, $N=8$ gauged supergravity, which describes $N=4$ SYM and most of its bilinear relevant operators (almost all of the masses for scalars and fermions). Or we may have an $N=4$ theory describing the orbifold R^4/Z_2 and the supersymmetric mass term that drives the theory to an $N=1$ IR fixed point. Or else we may have the Lagrangian for some of the KK modes. The interactions among the modes in the graviton multiplet in $5d$ can be found using supersymmetry. In particular, for the $N=4$ SYM case, the $5d$ Lagrangian for the *massless* modes

is uniquely fixed by supersymmetry in the form of the N=8 gauged supergravity [30]. All mass terms for the scalars and the fermions contained in the KK spectrum are associated to modes in the gauged supergravity. 5-dimensional supersymmetric Lagrangians have been discussed also for less supersymmetry, but the uniqueness of N=8 supergravity is lost and interesting modes are split into various vector, tensor and hypermultiplets. One needs some help from QFT intuition in identifying the right potential. In principle, $V(\lambda)$ can be obtained for all modes (often with non-trivial effort) by dimensional reduction from 10 dimensions.

If the UV CFT perturbed by a particular operator O_λ flows in the IR to another CFT, the potential V must have a critical point for non-zero value of the scalar field λ . Analogously, the dual of the flow to a non-conformal field theory is given by the flow from one minimum of the potential to infinity.

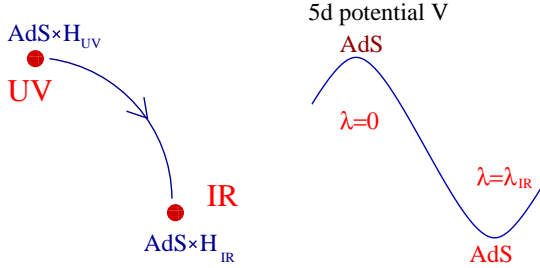


Figure 1: Schematic picture of the RG flow.

The 5d description of the RG flow between conformal theories is a kink solution, which interpolates between the two critical points. A 4d Poincaré invariant metric is

$$ds^2 = dy^2 + e^{2\phi(y)} dx^\mu dx_\mu, \quad \mu = 0, 1, 2, 3. \quad (2.2)$$

AdS corresponds to $\phi = y/R$. We then look for solutions with asymptotics: $\phi(y) \rightarrow y/R_{UV,IR}$ for $y \rightarrow \pm\infty$; $\lambda(y) \rightarrow 0$ for $y \rightarrow \infty$, while $\lambda(y) \rightarrow \lambda_{IR}$ for $y \rightarrow -\infty$. We associate larger energies with increasing y .

The equations of motion for the scalars and the metric read

$$\begin{aligned} \ddot{\lambda}_a + 4\dot{\phi}\dot{\lambda} &= \frac{\partial V}{\partial \lambda_a}, \\ 6(\dot{\phi})^2 &= \sum_a (\dot{\lambda}_a)^2 - 2V. \end{aligned} \quad (2.3)$$

With the above boundary conditions and a reasonable shape of the potential, a kink interpolating between critical points always exists [1].

As an example of flows between conformal field theories, we can discuss the mass deformations of N=4 SYM. These can be studied in the context of N=8 gauged supergravity, where the form of the potential V is known. N=8 gauged supergravity [30] is the low energy effective action for the “massless” modes of the compactification of type IIB on $AdS_5 \times S^5$. It is believed to be a consistent truncation of type IIB on S^5 in the sense that every solution of the 5d theory can be lifted to a consistent 10d type IIB solution. Five-dimensional gauged supergravity has 42 scalars, which transform under the N=4 YM R-symmetry $SU(4)$ as $\underline{1}, \underline{20}, \underline{10}$. The singlet is associated with the marginal deformation corresponding to a shift in the coupling constant of the N=4 theory. The mode in the $\underline{20}$ has mass square $M^2 = -4$ and is associated with a symmetric traceless mass term for the scalars $\text{Tr } \phi_i \phi_j$, ($i, j = 1, \dots, 6$) with $\Delta = 2$. The $\underline{10}$ has mass square $M^2 = -3$ and corresponds to the fermion mass term $\text{Tr } \lambda_A \lambda_B$, ($A, B = 1, \dots, 4$) of dimension 3. Thus the scalar sector of N=8 gauged supergravity is enough to discuss at least all mass deformations that have a supergravity description³.

The scalar potential V in eq.(2.1) is known and it turns out to have only isolated minima (apart from one flat direction, corresponding to the dilaton). Up to now, all critical points with at least $SU(2)$ symmetry have been classified [3]. There is a central critical point with $SO(6)$ symmetry and with all the scalars λ_a vanishing: it corresponds to the unperturbed N=4 YM theory. There are three N=0 theories with residual symmetry $SU(3) \times U(1)$, $SO(5)$ and $SU(2) \times U(1)^2$. They correspond to non-zero VEV for some of the scalars in the $\underline{10}$, $\underline{20}$, and $\underline{10} + \underline{20}$, respectively. Then there is an N=2 point with symmetry $SU(2) \times U(1)$, obtained giving VEV to scalars in the $\underline{10} + \underline{20}$ [3]. According to the AdS/CFT

³The only missing state is $\text{Tr } \sum_i \phi_i^2$, the prototype of a stringy states in the correspondence. Even without this state, we can study almost all massive deformations of the N=4 theory and all these deformations can be described by just the Lagrangian for the massless multiplet.

correspondence, these other minima should correspond to IR conformal field theories⁴. The following IR CFT theories can be obtained as mass deformations of N=4 SYM:

- Three N=0 theories with symmetry $SU(3) \times U(1)$, $SO(5)$ and $SU(2) \times U(1)^2$. All these theories are unstable and correspond to non-unitary CFTs. A natural question arises: are all the N=0 critical points unstable?
- A stable N=1 theory with symmetry $SU(2) \times U(1)$. It corresponds to the N=4 theory deformed with a mass for one of the three N=1 chiral superfields. Results and supergravity description [4, 8] are almost identical to the $T^{1,1}$ case, which is just a Z_2 projection of this example.

2.1 Central charges

In a supersymmetric gauge field theory in 4d, the trace and R-symmetry anomaly are given by [31]

$$T_\mu^\mu = \frac{\tilde{\beta}}{2g^2} F_{\mu\nu}^2 + \frac{c}{16\pi^2} W_{\mu\nu\rho\sigma}^2 - \frac{a}{16\pi^2} \tilde{R}_{\mu\nu\rho\sigma}^2 + \frac{c}{6\pi^2} V_{\mu\nu}^2 + \frac{b}{32\pi^2} B_{\mu\nu}^2, \quad (2.4)$$

$$\partial_\mu \sqrt{g} R^\mu = -\frac{\tilde{\beta}}{3g^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{a-c}{24\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} + \frac{5a-3c}{9\pi^2} V_{\mu\nu} \tilde{V}^{\mu\nu} - \frac{b}{48\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu} \quad (2.5)$$

Here $W_{\mu\nu\rho\sigma}$ and $R_{\mu\nu\rho\sigma}$ are the Weyl and curvature tensors for an external metric $g_{\mu\nu}$ that couples to the energy-momentum tensor $T_{\mu\nu}$. Similarly $V_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths of the external sources V_μ, B_μ that couple to the R-symmetry and flavour currents, respectively. $F_{\mu\nu}$ is the gauge field strength and $\tilde{\beta}$ is the numerator of the exact beta-function [32].

The external anomaly coefficients a and c have a straightforward interpretation in the dual supergravity theory.

c is the central charge of the CFT, and it is associated with the cosmological constant at the critical points. From eq. (2.1), we can see by a

⁴The symmetries of field theories can be read from those of the supergravity minima according to the correspondence : gauge symmetry in supergravity \leftrightarrow global symmetry in field theory, supersymmetry in supergravity \leftrightarrow superconformal symmetry in field theory.

simple scaling that, at least at the fixed points, where $ds^2 = R^2[dy^2 + \exp(2y) \sum_i dx_i^2]$,

$$\langle T(x)T(0) \rangle = \frac{c}{|x|^8} \rightarrow c \sim R^3 \sim (\Lambda)^{-3/2}. \quad (2.6)$$

This scaling reproduces the known results for c [27, 33]. More interestingly, one can prove that for the class of field theories that have a supergravity dual a c -theorem exists. Indeed we can exhibit a c -function that is monotonically decreasing along the flow [1, 8]. The c -function

$$c(y) \sim (T_{yy})^{-3/2}, \quad (2.7)$$

is constructed with the y component of the stress-energy tensor

$$T_{yy} = 6(\dot{\phi})^2 = \sum_a (\dot{\lambda}_a)^2 - 2V. \quad (2.8)$$

At the critical points, where $\dot{\lambda}_a = 0$,

$$c(y) = c_{UV,IR} \sim (-V)_{UV,IR}^{-3/2} \sim \Lambda_{UV,IR}^{-3/2}, \quad (2.9)$$

and using the equations of motion ($\ddot{\phi} < 0$) and the boundary conditions one can easily check that $c(y)$ is monotonic [1, 8].

Let us consider a . AdS computations [33] showed that $a = c$ for all CFTs that have an AdS dual.

It is then natural to ask what can AdS/CFT correspondence say about the coefficient b ⁵. The coefficient b is related to the two-point function of the flavour (global) symmetry currents [31]. According to AdS/CFT correspondence the R-symmetry and flavour currents are associated to the gauge fields of the SUGRA Lagrangian

$$J_\mu, R_\mu \longleftrightarrow A_\mu. \quad (2.10)$$

One should then be able to read the b (and a) coefficient from the kinetic terms of the corresponding SUGRA modes. The generic 5d-Lagrangian we are interested in has the following structure

$$L = \sqrt{-g} \left[-\frac{R}{4} + \Lambda + f F_{\mu\nu}^2 + f_R F_{\mu\nu R}^2 \right]. \quad (2.11)$$

Here $F_{\mu\nu R}$ and $F_{\mu\nu}$ represent the kinetic terms for the fields corresponding to the R-symmetry

⁵These results have been obtained in collaboration with D. Anselmi and L. Girardello.

and flavour symmetry currents, respectively. At the critical points (or generically for a metric of the form (2.2)), one obtains by scaling

$$\langle J(x)J(0) \rangle = \frac{b}{|x|^6} \rightarrow b \sim fR \sim fc^{1/3}. \quad (2.12)$$

A similar behaviour is obtained for the R-symmetry currents. In this case, supersymmetry⁶ implies $b = c$, and the previous equation can be used as a check of the consistency of the procedure.

The values of the coefficients f and f_R depend on the particular model under consideration. Consider for example the massive deformations of N=4 SYM, for which we have the dual supergravity Lagrangian: that of N=8 gauged supergravity. In this case, the kinetic term for the gauge fields is expressed in terms of the vielbein parametrising the scalar manifold [34]. To determine f and f_R we have then to evaluate the contractions of the vielbein and therefore these coefficients depend on the critical point and on the way the UV $SU(4)$ group is broken (for instance, $SU(4) \rightarrow SU(3) \times U(1)_R$, or $SU(4) \rightarrow SU(2) \times U(1)_R$, ...). We now want to compute the charge b for the global non-abelian symmetry group preserved along the flow (e.g. $SU(3)$, $SU(2)$, ...). The computation of the coefficients f can be performed using the results of [34] for most of the critical points. Alternatively, using the parametrisation in appendix A of [8], it is easy to convince themselves that

$$f = e^{4\alpha}. \quad (2.13)$$

Here α is the scalar in the 20 of $SU(4)$ corresponding to a mass term for the scalars in N=4 SYM [8]. The value of the scalar α and c for the various fixed points can be found in [3, 8, 34]. One then gets the following results for the coefficient b [35]:

- N=1 point with symmetry $SU(2) \times U(1)$. $\frac{b_{IR}}{b_{UV}} = \frac{3}{2}$. This is the only case where comparison with field theory is possible. Consider a set of N=1 chiral superfields X_i in the representation R_i of the gauge group and in the representation T_i of the flavour

symmetry group. Then, because of supersymmetry, the following formula holds [31]

$$b_{UV} - b_{IR} = 3 \sum_{ij} (\dim R_i) \left[\left(r_i - \frac{2}{3} \right) T_i^j T_j^i \right], \quad (2.14)$$

where r_i is IR R-symmetry charge of the field X_i and T_i^j are the generators of the flavour group in the representation T_i . It is straightforward to check that the supergravity and the field theory computations agree.

- N=0 theories. For the $SU(3) \times U(1)$, $SO(5)$ and $SU(2) \times U(1)^2$ symmetric points, we have $\frac{b_{IR}}{b_{UV}} = \frac{2\sqrt{2}}{3}$, $\frac{b_{IR}}{b_{UV}} = \sqrt{2}$ and $\frac{b_{IR}}{b_{UV}} = 2$, respectively.

In [36] it was observed that for several examples of supersymmetric gauge theory b increases going from the UV to the IR. This was suggestive of possible anti- b -theorem. The same authors however pointed out that for non-supersymmetric gauge theories b has no universal behaviour, and that also a large class of supersymmetric theories violates the relation $b_{IR}/b_{UV} > 1$. Then it is not possible to state any anti- b -theorem in field theory. It is interesting to see what are the supergravity results. Consider first the non-supersymmetric cases. For the point $SU(3) \times U(1)$ we have $b_{IR}/b_{UV} < 1$, which violates the anti- b -theorem. The situation is different for the supersymmetric point $SU(2) \times U(1)$. In this case the coefficient b increases along the flow. The same analysis carried on for the massive flow to N=1 super Yang-Mills (see section 4) or for the Coulomb branch of N=4 SYM [9] seems to indicate a similar behaviour.

Notice that the theories that have a supergravity dual represent a very restricted class of gauge theories. First of all these theories always have $a = c$, which is in general not the case in field theory. It has been argued that the requirement $a = c$ simplifies the structure and OPEs of a CFT, making it most similar to a two dimensional conformal field theory [37]. Secondly it has been suggested (see [8] and next section) that all these theories could be characterised by having a pre-potential. It could then be possible, and interesting to check, whether an anti- b -theorem

⁶The R-symmetry currents are in the same multiplet as the energy-momentum tensor.

could hold for this particular class of gauge theories.

The previous results on b could have been obtained from the analysis of the Chern-Simons terms of the $N=8$ Lagrangian, which contain all information about global anomalies [38, 39]. In particular, b can be read from the $SU(2)^2 \times U(1)_R$ anomaly coefficient, which can be extracted from the Chern-Simon terms. It is easy to check, using the results in [8], that the result for b coincides with the previously obtained one⁷. Notice that the Chern-Simon terms uniquely determine the form of a supersymmetric gauge supergravity. From the knowledge of the global anomaly, we should be able to reconstruct the entire AdS Lagrangian for *massless* modes for a given supersymmetric CFT fixed point [39].

2.2 Vacua and deformations

We end this section with a brief discussion of a point that will play an important role in our analysis, namely the fact that supergravity solutions can represent both deformations of a CFT and different vacua of the same theory [15, 16]. The running of coupling constants and parameters along the RG flow can be induced in the UV theory in two different ways: by deforming the CFT with a relevant operator, or by giving a nonzero VEV to some operators. The asymptotic UV behaviour discriminates between the two options. In the asymptotic AdS-region, we just need a linearised analysis. A scalar fluctuation $\lambda(y)$ in the asymptotically AdS background must satisfy

$$\ddot{\lambda} + 4\dot{\lambda} = M^2\lambda, \quad (2.15)$$

where the dot means the derivative with respect to y . The previous equation has a solution depending on two arbitrary parameters

$$\lambda(y) = Ae^{-(4-\Delta)y} + Be^{-\Delta y}, \quad (2.16)$$

where Δ is the dimension of the operator, $M^2 = \Delta(\Delta-4)$ [38, 40]. We are interested in the case of relevant operators, where $\Delta \leq 4$. From the basic prescription of the AdS/CFT, we associate solutions behaving as $e^{-(4-\Delta)y}$ with deformations of

⁷It is crucial to pay attention to normalisations and the definition of $U(1)_R$, which varies from UV to IR.

the $N=4$ theory with the operator O_λ . On the other hand, solutions asymptotic to $e^{-\Delta y}$ (the subset with $A = 0$) are associated with a different vacuum of the UV theory, where the operator O_λ has a non-zero VEV⁸[15, 16].

Since in general the UV-IR interpolating solution is not known, it is not even obvious whether a particular solution corresponds to a deformation or to a different vacuum. For many problems, we may invoke supersymmetry. It helps in finding the solution all along the flow and in unambiguously identifying the UV behaviour. In ref. [8, 19] the conditions for a supersymmetric flow were found. As usual, a solution for which the fermionic shifts vanish, automatically satisfies the equations of motion. Moreover, this shortcut reduces the second order equations to first order ones. For a supersymmetric solution, the potential V can be written in terms of a superpotential W as

$$V = \frac{1}{8} \sum_{a=1}^n \left| \frac{\partial W}{\partial \lambda_a} \right|^2 - \frac{1}{3} |W|^2, \quad (2.17)$$

where W is one of the eigenvalues of the tensor W_{ab} defined in [34]. The equations of motion reduce to

$$\dot{\lambda}_a = \frac{1}{2} \frac{\partial W}{\partial \lambda_a}, \quad (2.18)$$

$$\dot{\phi} = -\frac{1}{3} W. \quad (2.19)$$

It is easy to check that a solution of eq.(2.19) satisfies also the second order equations (2.3).

It is quite plausible and generally assumed that all the supergravity flows connecting fixed points correspond to deformations of the UV fixed point.

3. Confining Solutions

Solutions flowing to infinity represent RG flows to non-conformal theories, which may exist in various phases in the IR. These kinds of solution are difficult to classify. In many cases the asymptotic IR behaviour is known, but the entire solution along the flow can not be found. Typically,

⁸We are not careful about subtleties for particular values of Δ [16].

we encounter a singularity somewhere along the flow. Many solutions exhibit a logarithmic divergence at finite y_0 for the scalar fields, $\lambda_a \sim B_a \log |y - y_0|$, and the metric, $\phi \sim A \log |y - y_0|$. There are many criteria for studying the IR properties and the phase of these solutions. One of them, the Wilson loop, will be discussed later. The spectrum can be determined also from two-point functions, where physical bound states appear as poles. Poles in the two-point function corresponding to a minimally coupled scalar, for example, correspond to F^2 glueball masses in the field theory. The analysis of the spectrum can be reduced, as usual in the AdS/CFT correspondence, to the solution of a Schroedinger problem [22, 41]. After a change of variable $y \rightarrow z$ to the conformally flat metric $ds^2 = e^{2\phi(z)}((dz)^2 + (dx)^2)$ and a field redefinition $\Phi_k(z) = e^{-3\phi(z)/2}\psi(z)$, the 5d equation for a minimally coupled scalar $\Phi(x, y) = e^{-ikx}\Phi_k(y)$ takes the Schroedinger form

$$(-\partial_z^2 + V(z))\psi = E\psi \quad (3.1)$$

where $V = \frac{3}{2}\phi'' + \frac{9}{4}(\phi')^2$. The eigenvalues E give the poles in the two-point function and the spectrum.

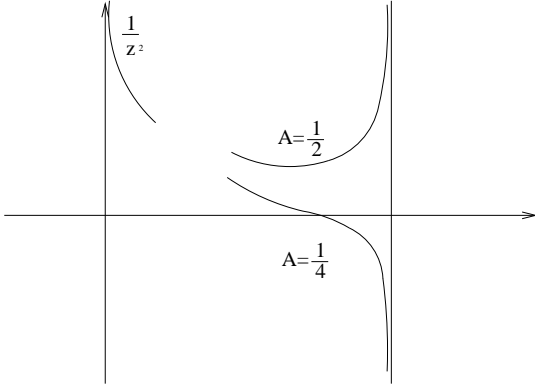


Figure 2: The Schroedinger potential in various cases.

The form of V immediately tells us whether the theory has a mass gap and a discrete spectrum or a continuous one, whether it confines or not. Unfortunately, in very few examples V is known along the entire flow. We can nevertheless extract some information from the IR behaviour. For the logarithmically divergent flows discussed above, if $A < 1$, the singularity is mapped to a

finite z_0 and we have

$$V \sim \frac{3A(5A - 2)}{4(1 - A)^2(z - z_0)^2}. \quad (3.2)$$

This behaviour looks potentially dangerous, but, as discussed in all quantum mechanics textbooks, $V \sim k/z^2$ has a discrete spectrum bounded from below, provided $k \geq -1/4$. It is easy to check that, for the logarithmically divergent flows, this condition is always satisfied. The value $k = -1/4$ is obtained for $A = 1/4$. This is the value that appears in many solutions where the supergravity potential is irrelevant in the IR [6], but also in one of the examples of N=4 coulomb branch in [9]. If $A > 1$, the singularity is mapped to $z = \infty$, the potential goes to zero and we may expect portions of continuous spectrum. Clearly, any sensible prediction about the spectrum requires the full knowledge of V . The same Schroedinger equation is to be considered when looking at generalisations of the RS scenario.

3.1 Supersymmetric and non-supersymmetric examples

We now briefly discuss few examples in the literature.

In [6], the class of non-supersymmetric solutions where the potential can be neglected in the IR have been discussed. They all have $A = 1/4$. It was argued that they may exhibit a variety of IR behaviours, from confinement to screening, depending on the values of the constants B_a . Since we can not follow the solution from UV to IR, it is difficult to make more meaningful claims. We do not even know whether these solutions correspond to deformations or to different vacua of the UV fixed point.

In the N=4 Coulomb branch solutions discussed in [9], A assumes various values. There is one solution with $A = 1/5$, one with $A = 1/4$ and all the other have $A > 1/4$. The UV behaviour can be unambiguously determined using the first-order equations (2.19). All these solutions correspond to different vacua (Coulomb branch) of the UV fixed point.

The supersymmetric massive flow from N=4 to N=1 SYM was discussed in [11]. It has $A = 1/2$. The qualitative properties of the solution

agree with QFT expectations. They are discussed in the next section.

Due to the IR singularity, not all the previous solutions are expected to be *physical*. A possible criterion for selecting the *physical* solutions has been proposed in [18]. According to this criterion, the supergravity potential must be bounded above along the flow. This seems to eliminate all solutions with $A < 1/4$. The case $A = 1/5$ in the examples of N=4 Coulomb branch is indeed known to correspond to a singular 10d solution with negative tension branes. The criterion can be also understood as follows. It selects solutions for which the IR ambiguities noticed in [6] are absent. The action for a (canonically normalized) scalar $S = \int e^{4\phi}(\partial\lambda)^2$ predicts an IR contribution to the condensate

$$\langle O_\lambda \rangle = \frac{\delta S}{\delta \lambda} \sim e^{4\phi} \partial\lambda \sim |y - y_0|^{4A-1} \quad (3.3)$$

for all logarithmic flows. This IR ambiguities diverges when $A < 1/4$. The case $A = 1/4$ is borderline. It is possible that, as noticed in [18], only the $A = 1/4$ solutions representing vacua have a *physical* interpretation.

4. The Flow to N=1 SYM

We now present a holographic RG flow from N=4 SYM to pure N=1 SYM in the IR. We find agreement with field theory expectations: quarks confine, monopoles are screened, and there is a gaugino condensate.

Consider a deformation of N=4 Super Yang-Mills theory with a supersymmetric mass term for the three fermions in the chiral N=1 multiplets. In N=1 notations, this is a mass term for the three chiral superfields X_i

$$\int d^2\theta m_{ij} \text{Tr } X_i X_j + \text{c.c.}, \quad (4.1)$$

where m_{ij} is a complex, symmetric matrix.

The theory flows in the IR to pure N=1 Yang-Mills, which confines. To obtain the standard N=1 pure Yang-Mills with fixed scale Λ , we need a fine tuning of the UV parameters, in which the mass m diverges while the 't Hooft coupling constant, x , goes to zero as an (inverse) logarithm of m . This is outside the regime of validity of supergravity, which requires a large x .

We can think of m as a regulator for N=1 SYM. When embedded in N=4 SYM, the theory is finite. To get a well defined N=1 SYM, we remove the cut-off ($m \rightarrow \infty$) with a fine tuning of the coupling ($x(m) \rightarrow 0$). However, if we use supergravity, we are in the large x regime. The massive modes have a mass comparable with the scale of N=1 SYM and they do not decouple. We can think of this as a theory with an ultraviolet cut-off. A good analogy is with lattice gauge theory. $1/m$ corresponds to the lattice spacing. The continuum limit is obtained with a fine tuning $a \rightarrow 0, g(a) \rightarrow 0$. However we can study the lattice theory at strong coupling, far from the continuum limit. A standard computation at strong coupling (by Wilson) gives the area law. We are just doing analogous computations with supergravity. Qualitative features of the theory should hold also at strong coupling.

The 5-dimensional action for the scalars [34]

$$L = \sqrt{-g} \left[-\frac{R}{4} - \frac{1}{24} \text{Tr} (U^{-1} \partial U)^2 + V(U) \right], \quad (4.2)$$

is written in terms of a 27×27 matrix U , transforming in the fundamental representation of E_6 and parametrising the coset $E_6/USp(8)$. In a unitary gauge, U can be written as $U = e^X$, $X = \sum_a \lambda_a T_a$, where T_a are the generators of E_6 that do not belong to $USp(8)$. This matrix has exactly 42 real independent parameters, which are the scalars of the supergravity theory. They transform in the following $SO(6)$ representations: $\underline{10}_c$, $\underline{20}$, and $\underline{1}_c$. The supersymmetric mass term for the chiral multiplets, m_{ij} , transforms as the $\underline{6}$ of $SU(3) \in SO(6)$, and the corresponding supergravity mode appears in the decomposition of the $\underline{10} \rightarrow \underline{1} + \underline{6} + \underline{3}$ of $SU(4)$ under $SU(3) \times U(1)$. The term $\underline{1}$ in this decomposition corresponds instead to the scalar σ dual to the gaugino condensate in N=1 SYM. In principle, a generic non-zero VEV for m_{ij} will induce non-zero VEVs for other scalars as well, due to the existence of linear couplings of m to other fields in the potential. However, if we further impose $SO(3)$ symmetry by taking m_{ij} proportional to the identity matrix, a simple group theory exercise shows that all the remaining fields can be consistently set to zero. This is true also if we consider a two-parameter

Lagrangian depending on both m and σ . This felicitous circumstance makes an apparently intractable problem very simple and exactly solvable.

The actual computation is reported in [11]. The result for the action for m and σ (the reason why we are considering both modes will be clear very soon) is

$$L = \sqrt{-g} \left\{ -\frac{R}{4} + \frac{1}{2}(\partial m)^2 + \frac{1}{2}(\partial \sigma)^2 + \right. \\ \left. -\frac{3}{8} \left[\left(\cosh \frac{2m}{\sqrt{3}} \right)^2 + 4 \cosh \frac{2m}{\sqrt{3}} \cosh 2\sigma \right. \right. \\ \left. \left. - (\cosh 2\sigma)^2 + 4 \right] \right\}. \quad (4.3)$$

The action has the supersymmetric form (2.17) with $W = -\frac{3}{4} \left(\cosh \frac{2m}{\sqrt{3}} + \cosh 2\sigma \right)$. The first order equations (2.19) read

$$\dot{\phi} = \frac{1}{2} \left(1 + \cosh \frac{2m}{\sqrt{3}} \right) \quad (4.4)$$

$$\dot{m} = -\frac{\sqrt{3}}{2} \sinh \frac{2m}{\sqrt{3}}, \quad (4.5)$$

$$\dot{\sigma} = -\frac{3}{2} \sinh 2\sigma. \quad (4.6)$$

One interesting feature of the solution is that the equations can be analytically solved. To the best of our knowledge, there is only another example of analytically solvable flow, describing the Coulomb branch of N=4 SYM [9]. The solution in our case is:

$$\phi(y) = \frac{1}{2} \log[2 \sinh(y - C_1)] + \\ + \frac{1}{6} \log[2 \sinh(3y - C_2)], \quad (4.7)$$

$$m(y) = \frac{\sqrt{3}}{2} \log \left[\frac{1 + e^{-(y-C_1)}}{1 - e^{-(y-C_1)}} \right], \quad (4.8)$$

$$\sigma(y) = \frac{1}{2} \log \left[\frac{1 + e^{-(3y-C_2)}}{1 - e^{-(3y-C_2)}} \right]. \quad (4.9)$$

The metric has a singularity at $y = C_1$ with $A = 1/2$

$$ds^2 = dy^2 + |y - C_1| dx^\mu dx_\mu. \quad (4.10)$$

Around this point m behaves as

$$m \sim -\frac{\sqrt{3}}{2} \log(y - C_1) + \text{const.} \quad (4.11)$$

Here we assumed that $C_2 \leq 3C_1$, so that at the point where m is singular, σ is still finite.

Let us notice that this intuitive criterion for selecting *physical* solutions is in agreement with the one proposed in [18], which exactly selects the solutions with $C_2 \leq 3C_1$. For $C_2 > 3C_1$, σ diverges first with a value $A = 1/6$. For these and other reasons, we regard these solutions as *unphysical*.

4.1 Properties of the solution

Let us discuss the qualitative properties of the N=1 SYM solution.

It is easy to see that the solution corresponds to a true deformation of the gauge theory. Indeed, m approaches the boundary in the UV ($y \rightarrow \infty$) as $m \sim e^{-y}$, which is the required behaviour of a deformation (see eq.(2.16)). On the other hand, σ has the UV behaviour appropriate for a condensate $\sigma \sim e^{-3y}$. Let us stress that this behaviour is enforced by the requirement of N=1 supersymmetry along the flow. The interpretation of the solution is therefore the following: upon perturbation with a mass term for the three chiral fields, the N=4 SYM theory flows in the IR to pure N=1 SYM in a vacuum with a non-zero gaugino condensate. The existence of a gaugino condensate is one of the QFT expectations for N=1 SYM.

We also expect the gauge theory to exhibit confinement in the IR. We can easily compute a two-point function for a minimally-coupled scalar in the background with $\sigma = 0$. In our example, the Schroedinger potential is

$$V(z) = \frac{6 \cos(2z) + 9}{\sin^2(2z)}. \quad (4.12)$$

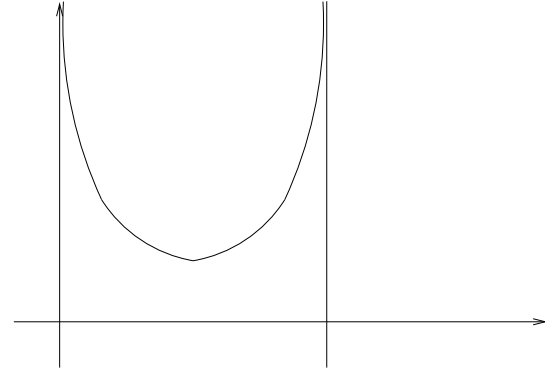


Figure 3: The potential for the N=1 SYM flow.

It is obvious from the figure below that there is mass gap and a discrete spectrum. The AdS boundary is at $z = 0$ and the singularity at $z = \pi/2$.

The two-point function for the massless scalar corresponding to F^2 can be explicitly computed [17]:

$$\langle F^2(k)F^2(0) \rangle \sim k^2(k^2 + 4)\text{Re} \psi(2 + ik). \quad (4.13)$$

It approaches the conformal expression $k^4 \log k$ in the UV and it is analytic for small k , as appropriate for a confining theory. It has poles for $M^2 = -k^2 = n^2, n = 2, 3, \dots$, corresponding to the F^2 glueball states in the spectrum.

Despite the presence of a singularity that invalidates the supergravity approximation in the IR, the qualitative properties of the solution agree with the QFT expectations. There is however a disturbing point: our solution depends on two independent parameters C_1 and C_2 . The first one fixes the position of the singularity and it is related to the magnitude of the mass deformation. The second one is instead related to the magnitude of the gaugino condensate. We have a chirally-symmetric vacuum and, more disturbing, a continuous degeneracy of vacua with arbitrary small condensate. We certainly expect that the correct treatment of the singularity and its resolution in string theory fixes the relation between C_1 and C_2 in agreement with field theory expectations. We do not still know how to resolve or deal with the singularity, therefore we limit ourself to a brief discussion of the QFT expectations and possible interpretations of the singularity.

4.2 QFT and string expectations

Strong coupling QFT results for N=1 SYM have been recently obtained and differ considerably from the weak coupling ones [43]. At weak coupling, spontaneous breaking of the chiral symmetry Z_N gives N vacua that only differ for the phase of the gaugino condensate $\langle \lambda\lambda \rangle \sim e^{2\pi i k/N} \Lambda_{\text{N=1}}^3$. In the large N limit, we obtain a *circle* of vacua. The magnitude of the gaugino condensate is fixed in terms of the SYM scale $\Lambda_{\text{N=1}} \sim m e^{-1/3Ng^2}$. At strong coupling instead, it was shown in [43] that there is, at least for $\theta = 0$, a distribution of

vacua with condensate $\langle \lambda\lambda \rangle \sim m^3 x^3 / j^2$, $j = 1, 2, \dots$ with zero phase. The weakly coupled *circle* is lost, the condensate magnitude is not fixed and the vacua have an accumulation point at the origin (zero condensate). However, we notice that the structure of vacua found in [43] has many similarities with our supergravity result. As independently noticed in [18], it is tempting to identify the solution with $C_2 = 3C_1$ with the $j = 1$ vacuum in [43]. The other solutions with $C_2 < 3C_1$ should correspond to the $j \neq 1$ vacua. To see how the continuum of vacua in supergravity is reduced to a discrete numerable set, we should understand how to include string corrections in our computation. Notice that the solution with $\sigma = 0$, which is not appealing on the ground of weak coupling intuition, could be nevertheless used as a (reasonable?) approximation for the many vacua with small condensate at strong coupling.

It was also proposed in [18] to fix the relation between C_1 and C_2 by considering the finite temperature version of our solution, where conditions to be imposed at the horizon fix the parameters. One finds $C_2 = 3C_1$. This is the only special value for our parameters, since, exactly for $C_2 = 3C_1$, the two scalars m and σ diverge at the same point in y . In SYM the breaking of supersymmetry will select the vacuum with minimal energy. At weak coupling, where all the vacua have a condensate with the same magnitude, this procedure should give us also the value of the N=1 condensate. At strong coupling, with condensates of almost arbitrary magnitude, this would give information at most about one particular vacuum ($j = 1$?).

The knowledge of the full 10 dimensional solution would greatly help in understanding the properties of the RG flow and in studying possible resolutions of the singularity. It may even happen that the singularity is an artifact of the dimensional reduction, that disappears in 10d. This happens, for example, in the case of the Coulomb branch of N=4 SYM [9], where the 10 dimensional background is just a regular continuous distribution of D3-branes. However, even in this context, some other equally nice⁹ 5d so-

⁹But not satisfying the criterion in [18].

lutions have a lift to still singular $10d$ solutions, representing D3-branes with negative tension. The complete ansatz for the $10d$ lifting of $5d$ solutions is known only for a subset of scalars, the 20, coming from the KK modes of the internal metric. This is sufficient to lift all solutions representing the Coulomb branch, but it is not of help with our solution, where the modes 10 from the anti-symmetric tensors are excited.

A ten dimensional interpretation of the N=1 solution in terms of a background with also D5-branes has been proposed in [42]. We only notice that the ingredients in this interpretation (D5 and NS-branes) have been independently suggested in [43] on the basis of the strong coupling QFT analysis.

Finally, we mention that a mechanism for resolving singularities in distributions of branes which may help, after the $10d$ lifting, has been proposed in [21].

4.3 The Wilson loop

A complementary approach for checking confinement is the computation of a Wilson loop, which should manifest an area law behaviour. We need to minimise the action for a string whose end-points are constrained on a contour C on the boundary. The detailed computation is reported in [6, 11]. In the coordinates used in those papers, the quark-antiquark energy reads

$$E = S/T = \int dx \sqrt{(\partial_x u)^2 + f(u)}. \quad (4.14)$$

where $f(u) = T^2(u)e^{4\phi(u)}$. The phase of the theory can be inferred by the IR behaviour of this function (see [6] for a review of the various cases). $T(u)$ is the tension of the fundamental (in the case of a quark loop) or of the D1 string (monopole) in five dimensions. They are in general non-trivial functions of the scalar fields. The $5d$ N=8 gauged supergravity has an $SL(2, Z)$ symmetry that allows to discriminate electric and magnetic strings. They should couple to the $5d$ antisymmetric tensors $B_{\mu\nu}^{I\alpha}$, transforming in the $(\underline{6}, 2)$ of $SO(6) \times SL(2, Z)$. The $SO(6)$ index should account for the orientation of the strings on the five-sphere, while the $SL(2, Z)$ index should identify electric and magnetic quantities. On the basis of naive dimensional reduc-

tion from ten dimensions, the tensions can be read from the coefficients of the kinetic term for the antisymmetric tensors. In 10 dimensions, the tension of the fundamental string (or the D1-string) can be read from the NS-NS (or R-R) antisymmetric tensor Lagrangian evaluated in the Einstein frame,

$$\frac{1}{T_{F1}^2} H_{NS-NS}^2 + \frac{1}{T_{D1}^2} H_{R-R}^2. \quad (4.15)$$

A simple Weyl rescaling shows that this property is valid also in the five-dimensional theory in the Einstein frame.

The kinetic terms for the anti-symmetric tensors can be computed for the N=1 SYM solution and behave asymmetrically in the $SL(2, Z)$ indices [11]. The final result for the tensions $T(u)$ of the fundamental strings and of the D1-strings are, respectively,

$$T_{F1}^2 = 4 \left(\cosh \frac{4m}{\sqrt{3}} + \cosh \frac{2m}{\sqrt{3}} \right), \quad (4.16)$$

$$T_{D1}^2 = 8 \left(\cosh \frac{m}{\sqrt{3}} \right)^2, \quad (4.17)$$

so that the asymptotic behaviour of the corresponding functions $f(u)$ is

$$f_{(q\bar{q})}(u) \sim 1, \quad f_{(m\bar{m})}(u) \sim |u - C_1|. \quad (4.18)$$

It is easy to check that $f_{(q\bar{q})}(u)$ is bounded from below. It follows that the energy $E \geq cL$, where L is the quark distance. It can be easily proven that it is in fact $E = cL$, implying an area law behaviour for the Wilson loop, as expected for a confining theory. The IR behaviour of $f_{(m\bar{m})}(u)$ implies, on the other hand, that monopoles are screened (see [6] for a review).

There is an apparent contradiction in the previous reasoning. The $5d$ dilaton is not running in our solution. If the $10d$ dilaton were also constant, the tension for a fundamental string would be proportional to the tension of a D1-string and the same would be true also after dimensional reduction to 5 dimensions. The $5d$ tensions would be then complicated functions of the scalars, but invariant under $SL(2, Z)$. We instead find an $SL(2, Z)$ asymmetric result from the N=8 gauged supergravity evaluated along our

solution. A possible way out is to assume that, against naive expectations, the $10d$ dilaton is not constant. Clearly, it also exists the option that the $10d$ dilaton is constant and that the argument which determines the $5d$ tensions via dimensional reduction is too naive. However, we are not aware of any argument that rules out the possibility of a running $10d$ dilaton. Since we are not expert in reconstructing $10d$ solutions from $5d$ ones, we just limit ourselves to consider this option and perform some very preliminary check on the equations of motion.

The $10d$ dilaton equation of motion is

$$\partial^2 \phi \sim G_{MNP} G^{MNP}. \quad (4.19)$$

Therefore, a non-vanishing anti-symmetric tensor is a source for the dilaton. We can perform a check on our solution at the linearised level. Consider a generic fluctuation of the anti-symmetric tensor $B_{ab} = f_I(y) Y_{[ab]}^{I\pm}$. We refer to [44] for notations and useful equations. Here $Y_{[ab]}^{I\pm}$, $a, b = 1, \dots, 5$ are harmonic functions on the five-sphere, transforming in the representation I of $SO(6)$. They satisfy $\epsilon_{abcde} \partial_c Y_{[de]} = \pm 2i(k+2) Y_{[ab]}$, where k is an integer labelling the harmonic degree. It is then easy to check that

$$\partial^2 \phi \sim \frac{1}{3} ((\partial_y f)^2 - (k+2)^2 f^2) Y_{[ab]} Y_{[ab]}. \quad (4.20)$$

In our case ($I = \underline{10}$) $k = 1$. Since we are considering a deformation of the UV fixed point, $f \sim e^{-x}$, we see that the dilaton must run. Notice that instead, considering a different vacuum of the UV theory, one has $f \sim e^{-3x}$, and the dilaton remains constant (at least at the first perturbative order).

We still need to check that $Y_{[ab]} Y_{[ab]} \neq 0$. There is at least one example where $Y_{[ab]} Y_{[ab]} = 0$: the $SU(3) \times U(1)$ critical point of the $N=8$ supergravity, whose $10d$ solution is explicitly known [45]. In the product $\underline{10} \times \underline{10} = \underline{20} + \dots$, only the indicated term contains scalar terms ($SO(5) \in SO(6)$ singlets). It is easy to check that, decomposing $\underline{10} = \underline{1} + \underline{3} + \underline{6}$ under $SU(3) \times U(1)$, the $\underline{1}$ term (related to the $SU(3) \times U(1)$ critical point) has vanishing square. The $N=1$ mass term $\underline{6}$, however, has non vanishing square.

This argument is certainly not a proof that the $10d$ dilaton runs. However, we find this option appealing. A running of the $10d$ dilaton

would agree with an interpretation of our solution that includes branes others than the D3s. In many respects, the knowledge of the explicit $10d$ solution would help us in understanding the system, from the constituent branes to the fate of the singularity. Using a D3-brane probe in the $10d$ background we could also explicitly compute the running of the gauge coupling along the flow.

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